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Knowledge Growth: Applied Models of General and Individual Knowledge Evolution

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ABSTRACT

The article considers the mathematical models of the growth and accumulation of scientific and applied knowledge since it is seen as the main potential and key competence of modern companies. The problem is examined on two levels - the growth and evolution of objective knowledge and knowledge evolution of a particular individual. Both processes are described mathematically by exponential and logistic laws and parameters (intensity of knowledge obsolescence and the knowledge half-life period) allowing application of the models to real practise of knowledge management.

KEYWORDS
knowledge growth; knowledge obsolescence; knowledge management

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Introduction

The idea of knowledge being particularly valuable and important for economic agents and economy in general, no longer needs to be confirmed by new facts and figures. Nowadays the knowledge economy is not just a term describing the new milestone of global economic development, but has very real forms – it is revealed by the rapid development of high-tech industries becoming main for regions and countries, by the formation of new forms of co-interaction between real and educational sectors, by social changes – by strengthening the idea of the value of knowledge in society. Along with the formation of knowledge economy to the real socio-economic structure its applied disciplines and directions are being developed and one of priority directions today is knowledge management. Registration and systematization of experience, procedures and knowledge management practices in the new direction of management underlines the demand for mechanisms of storage, administration and dissemination of knowledge in companies and professional communities to meet the challenges of modern information society and the pace of its development. Which in its turn provides the scientific and business communities with a range of actual real practice challenges, especially since the conceptual and theoretical foundations

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of the knowledge economy, dating back to the pioneering works of F. Machlup, give us powerful basis for their solving.

One of the main tasks of knowledge management in enterprises is to maintain corporate knowledge (consisting of knowledge of individual employees) up to date for solving professional problems and tasks. The need to maintain intellectual assets at the appropriate level for a modern company is dictated by the accelerating pace and the vector of development of an industry in which it operates, and the methods and forms of dissemination of knowledge among staff have gone far beyond the formal events. As a consequence, the construction of a reasonable system of training and dissemination of knowledge in companies, built in accordance with the vector of development of the industry, is topical and priority management knowledge task. The assessment of how, when, to which intensity, to what extent and direction to conduct training of employees - topical issues of knowledge management that require adequate justification and confirmation by the concept of numbers.

As a first approximation, the content, nature and direction of activities for corporate knowledge management must be aligned with the intensity relevant the industry knowledge is being replaced. The form of knowledge dissemination should also be taken into account and it is not always limited to the formal procedures of teaching and learning. It is shown (Gusakov, 2012; Glukhov, Korobko and Marinina, 2003; Dudelin and Kazakova, 2010) that the skills, experience and tacit knowledge often spread in the course of prolonged interpersonal communication. So the modern high-tech companies, in which the knowledge is a key resource, are facing the task of implementation of knowledge management mechanisms oriented on intensive updating of knowledge not limited by only formalized procedures for their distribution.

Knowledge, as it is, is developing according to its own logic - it is objective in its nature and does not depend on individuals. At the same time individual knowledge is subjective, as embedded in the structure of his personal perception, emotions, experiences, beliefs, prejudices and views. Subjective knowledge is always unique and determined by cognitive and emotional context. The nature of the concept of knowledge and the variety of its interpretations force us to consider the evolution of knowledge from different perspectives. In the context of enterprise knowledge management, we propose to divide the process in two levels complementing each other - such as the growth of general knowledge and knowledge accumulation by individuals.

The methodology and results of the study

General trends of scientific knowledge growth are subject to an exponential law, this fact is justified theoretically and supported by multiple studies of scientific activity indicators. Analysis of various metrics of scientific knowledge growth (Russian innovation index, 2011) showed that the "primary" indicators of it (volume of investment and involved scientific personnel, arrays of publications) and "derivatives" of these parameters (inventions, discoveries, innovations) are subject to the law of exponential growth. Many researchers across the world proved that study. For example, D. Price (1966) saw the priority factor of knowledge exponential growth in the development of communication and information transfer. Yablonsky (1977), who recorded model form of scientific knowledge growth, argues its exponential character by two reasons. Firstly, the exponential growth is a very common regularity of the dynamics of any human activity results, including research. Secondly, the exponential law adequately reflects the mechanism for the generation of scientific knowledge (Yablonsky, 1977). Formally, this is represented as follows:

$$\Delta X = r \cdot X_0 \qquad (1)$$

where – the knowledge available at time t_0; ΔX – knowledge increment at the time interval (t_0, t_1); r - coefficient of proportionality, the numerical value of which depends on the branch of science.

A distinctive feature of knowledge opposed to information is association with individuals who not only keep it, but also actively uses. For particular individuals the knowledge accumulation phenomenon acquires its own characteristics and laws. For example, Makarov (2009) pointed out on the subjective nature of knowledge. His agent-based model takes into account the factor of forgetting knowledge in the course of time ("weathering") and the division of roles in the dissemination of knowledge. A number of other models, in particular Ratner (2010), introduced the metric measurement of knowledge of individuals and cognitive distances between them, focusing on the quantitative aspects of the process to the detriment of quality.

Meanwhile, the phenomenon of knowledge and competencies accumulation by a person, the study of its qualitative aspects and identification of patterns are very relevant and are in demand for modern and high-tech knowledge companies. Along with the way global trends in an industry knowledge development defines general strategy outline for a company, the laws of the individual acquisition of knowledge of its employees enable to assess, monitor and predict the intellectual capital and the company's ability to solve certain problems with it.

The analysis of the existing scientific and methodological groundwork (Bakanova, 2015) shows that today's mathematical models representing the process knowledge accumulation by individuals are poorly implemented and represented mainly by heuristic descriptions. Many are based on empirical data and the existing approaches take into account the predominantly superficial, quantitative manifestations of the process.

Based on this, we propose an approach to solve the indicated actual scientific and applied problems that extends and complements the existing researches in this area.

Model of general knowledge growth and rate of its obsolescence

Presented by Yablonsky (1977) the general analytical expression of the growth of scientific knowledge reflects the dependence of the total knowledge on time and allows us to calculate the volume of scientific knowledge at the time t_1:

$$X_1 = X(t_1) = X_0 + \Delta X = X_0(1 + r) \qquad (2)$$

And more generally, after n time intervals:

$$X_n = X(t_n) = X_0(1 + r)^n \qquad (3)$$

Technically, this mathematical dependence works similar to the known in financial mathematics discounted cash flow formula, and allows to calculate the amount of knowledge in a given time on the basis of the original volume and growth factor. However, unlike the present value of future money reduced to date, in order to analyze the evolution of knowledge it is more useful to determine the proportion of today's knowledge the in next time period. This is consistent with the established in previous researches (Price, 1966; Yablonsky, 1977) causes and patterns of scientific knowledge growth, with the postulate that new knowledge is to lose its relevance over time.

Thus, the knowledge available at the initial time t_0 will be the share of knowledge existing at the moment of time t_n:

$$X_0 = \frac{1}{(1+r)^n} X_n \quad (4)$$

The lattermost is an analytical expression of knowledge sensitivity to time and a reflection of knowledge growth as such, which is consistent with the existing in modern knowledge management practices unit of knowledge obsolescence - its half-life period.

By analogy with natural science concept of half-decay of a radioactive substance half-life of knowledge is treated as a time after completion of training, during which the experts lose half the original competence (Shafranov-Kutsev, 2010). In other words, it is a period for an industry knowledge to lose half of its relevance being replaced by new knowledge. The measurement knowledge half-life is based on the core metrics of scientific activity: the number of publications, the number of scientists working in a particular field and others. The bibliometric analysis of the knowledge half-life in various fields of applied science (Price, 1966; Mikhailov, Chernyi and Giliarevskii, 1976) revealed the following characteristic of this process: the rate of obsolescence of knowledge is low enough for the humanities and is very high in the exact sciences and engineering.

The values of knowledge half-life for different branches of science allows to calculate the parameter r from (4), which may be applied for the analysis of speed of science development and can be interpreted as the rate of obsolescence of knowledge or knowledge replacement share. The values for the knowledge half-lives in various branches of science and calculated for these sectors knowledge obsolescence intensity indicators are given in Table 1.

Table 1. Half-life periods and intensity of knowledge obsolescence in some sciences

Branch of science	Half-life period, years	The intensity of knowledge obsolescence, %
Nuclear physics	5	15
Biology	10	7
Humanities	25	3
Information & Communication Technologies	4	19

The following analytical dependence is the continuous analogue of the discrete form of the knowledge obsolescence:

$$X(t) = X_0(1+r)^t = X_0 e^{\ln(1+r)^t} = X_0 e^{t\ln(1+r)} \Rightarrow X(t) = X_0 e^{\delta t} \quad (6)$$

or $$X_0 = e^{-\delta t} X(t) \quad (7)$$

where e - continuous rate of the knowledge obsolescence, showing how fast current knowledge "subside in time" (obsolete).

In terms of corporate knowledge management strategy, the knowledge obsolescence intensity gives information about the speed of knowledge update in a particular area.

The model of knowledge accumulation by individuals

The way of general industry knowledge development determines the urgency and demand of the knowledge of its owners. As a first approximation the rate of the knowledge growth intensity can be interpreted as a minimum share of the annual renewal of employees' knowledge. However, more accurate analysis requires an understanding of the general laws of knowledge accumulation by individuals. Consistent with the way the intensity of growth and, accordingly, obsolescence of knowledge are different in various fields, learning and updating of knowledge also requires appropriate for each area time-consuming. Since there is personalized knowledge in question, the role of the subjective knowledge in this process can be decisive but, however, in our opinion, from the perspective of corporate knowledge management it is enough to identify common patterns of accumulation, "aging" of knowledge among employees.

At first stage of the analysis of knowledge accumulation and obsolescence processes in companies it is necessary to implement knowledge assessment methodology, allowing to make measurements of the level of knowledge. We will not dwell on the principles and techniques of knowledge measurement (different approaches presented in Makarov, 2009, Pipiya, 2008, Silkina, 2014), but proceed to the consideration of the laws of knowledge accumulation by individuals.

Formalized representation of learning process in organizations established practice is assigning different categories and qualifications to employees depending on the level of their expertise in a particular area. Employee's qualification determines their ability to solve problems of a certain level and to apply their knowledge, skills and experience in their daily practice. Today's high-tech industries (and companies respectively) have settled their own terms and requirements for skill levels determination, unified and standardized. Different degrees of knowledge assimilation are laid in the modern educational standards, including "know – be able - own" graduation, extending the usual numerical assessment of knowledge levels of graduates.

The mathematical interpretation of qualification growth dynamics of employees in high-tech industries will provide reasonable numerical indicators for ownership levels of professional knowledge, the average time on its uptake, which in turn serves as the basis for knowledge management planning and organization in companies.

Let $X \in [0,1]$ be an assessment of possession degree of specific industry knowledge, then the equation characterizing the degree of ownership of the knowledge is obtained from the exponential growth equation (1) or its continuous analogue:

$$\frac{dx(t)}{dt} = lx(t) \qquad (8)$$

with the following considerations. Equation (8) describes the growth of a certain value according to its reached level and is not limited by any restrictions. However, when it comes to the degree of knowledge assimilation process, it is evident the presence of the upper limit being "1", corresponding to the total level of knowledge absorption. The growth constraints factors lead to the assumption that the growth rate of l is not constant, depending on the branch of science, but determined by the level of received knowledge x: $l = l(x)$. This function decreases as x increases by getting closer to the limit due to the exhaustion of growth reserves.

In case of linear dependence of the $r(x)$ on the accumulated volume of knowledge $l(x) = 1 - x$ the differential equation of individual knowledge evolution takes the following form:

$$\frac{dx(t)}{dt} = x(t)(1 - x(t)) \quad (9)$$

It is a well-known equation of logistical dynamics for which general solution can be written as:

$$x(t) = \frac{Ce^t}{1 + Ce^t} = \frac{C}{C + e^{-t}} \quad (10)$$

as well as the private:

$$x(t) = \frac{x_0}{x_0 + e^{-t}(1 - x_0)} \quad (11)$$

Where $X_0 \in (0, 1)$ – initial (minimum) level of knowledge acquired by an individual, fixed as the fact of having this knowledge.

Note that the logic of the assimilation and accumulation process of professional knowledge by individuals is consistent with the logistic curve dynamics (9) and the properties of its equation solution. Thus with the passage of time knowledge of employees can only accumulate, so *x(t)* is a monotonically increasing function throughout the region of its definition.

In the first stage of acquaintance with subject area individuals accumulate knowledge fairly slow, but with increasing speed. As the rate of individual knowledge grows the speed of its accumulation increases proportionally to its accumulated value (the value *x(t)*).

On the other hand, the proportionality of the derivative $\frac{dx(t)}{dt}$ to the difference $(1 - x(t))$ means slowing amount of knowledge learned as it approaches the saturation limit, when all the main points of the knowledge already learned by an individual.

There are "tipping points" in the geometry of the logistic curve, which being applied to the analysis of the learning process are interpreted as turning points of the dynamics of knowledge assimilation of by individuals.

Essentially these points define the transition of quantitative to qualitative changes when the learned knowledge becomes sufficient to solve professional problems at higher levels. In terms of knowledge management, these transitions mean raising of qualifications and grades, adopted by training procedures in a company.

In its geometry the logistic curve has a bend point corresponding to the value of x=0.5, and two points of maximum curvature, dividing the curve into segments, which are characterized by its dynamics curve.

The points of maximum curvature of the function graph are calculated by known formulas which in the case of the knowledge growth equation show values of knowledge accumulation thresholds - the time that is required at this stage of study and the corresponding proportion of the total amount of knowledge.

Identifying the key points of personalized knowledge growth dynamics seems to us an urgent task of knowledge management. For its solvation it is enough to have information on time spent by employees on average for accumulation of a half of knowledge amount.

The analysis of materials of the high-tech company done by Bakanova et al. (2015) shows that its employee need one and a half year on average for mastering of the half of professional field of knowledge. This value corresponds to the inflection point of the logistic curve, which in it turn is a change of convexity direction of the rate of growth and the accumulation of knowledge, and means that for assimilation of 50% of knowledge of the subject area employees spend an average of 1.5 years. The further statistical analysis of the knowledge accumulation curve showed that in the initial stage of learning an individual need to gain 12% of knowledge, which will become "the foundation" for future development of high-grade professional knowledge. This value is the "entry point" into a profession that requires certain knowledge, skills and abilities. This initially accumulated knowledge value is enough for intensive further mastering of knowledge. Thus, the steps in the process of learning the craft of professional match: 50% in 1.5 years, 92% for 2.25 years (see Figure 1).

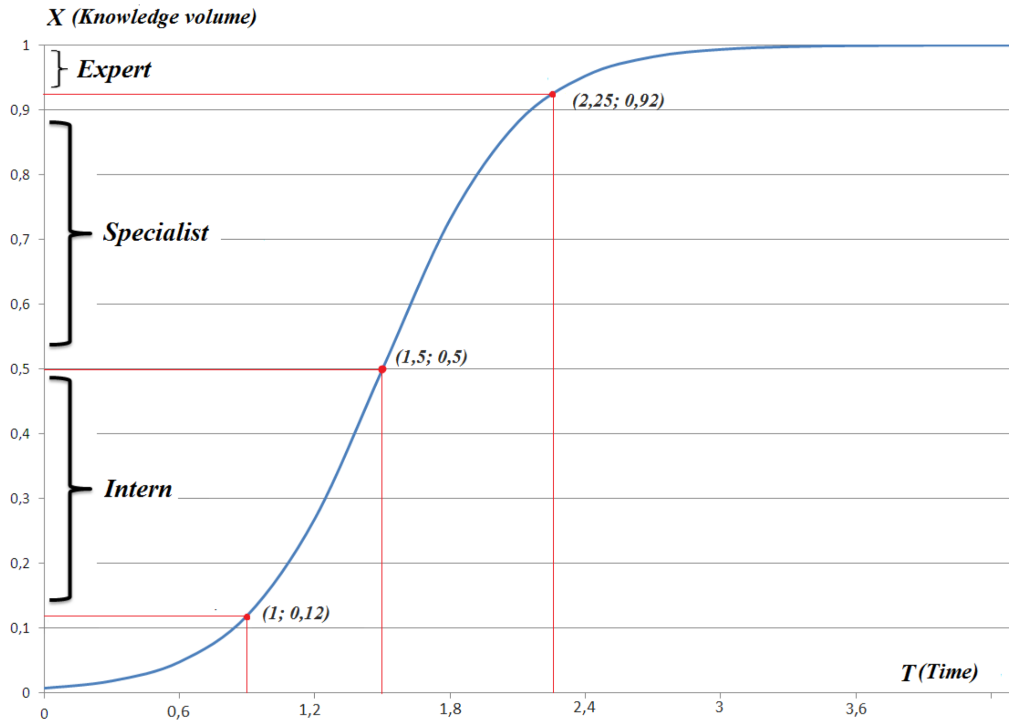

Figure 1. Graph of knowledge accumulation by employees.

The considered logistic law of knowledge accumulation dynamics reflects the subjective process of knowledge growth regardless of general context of subject area development, i.e. does not include the objective component of knowledge development. As it was shown, for a variety of scientific and applied knowledge is characterized by its speed of obsolescence of knowledge that advances the demand for continuous updating of staff knowledge.

In mathematical terms the two processes - assimilation of knowledge by an individual and the obsolescence of knowledge in general, is presented by the dependence:

$$x(t) = \frac{x_0}{x_0 + e^{-t}(1 - x_0)} e^{-\delta t} \qquad (12)$$

which reflects the level of knowledge of the specific individual based on actual state of general knowledge – the subjective process of knowledge accumulation is adjusted to general pace of knowledge development.

The correction to a factor of knowledge obsolescence allows to combine the two processes of knowledge growth - subjective and objective, and from a practical point of view - to understand how organization knowledge "subside" over time. So, for the above example which demonstrates that in general it takes 1,5 years for an employee to master the domain knowledge, it will be only 42% of up-to-date employee knowledge over that period of time. And for the same field of study the period for expert knowledge to "weather" and their skills to go to the previous level, is approximately 2 years.

Practical potential of the tool

Continuity of education and access to knowledge resources - one of the main trends of knowledge economy, leading to intensive and continuous intake and updating knowledge. The answer to this challenge for modern high-tech companies can be an organization of knowledge management being capable of maintenance of their intellectual capital of in up-to-date state. Building a knowledge management strategy, definition of its models, tools and practices, translated into scientific outline in the form of scientific and applied problems constitute, in our opinion, the most urgent open questions of modern management knowledge. The article contains some results of finding a solution to these problems, focusing, primarily, on the decision of applied problems in organizations such as distribution, storage and generation of knowledge in companies. Thus the discussed approaches to assessing the growth of general knowledge and the accumulation of individual one provide opportunities for organizations and systems to correct their procedures of knowledge management.

Like any other management process, the knowledge management at a first approximation involves an assessment of the current state of a system, determination of its desired state and a set of actions to achieve it. And in the context of the global mission of knowledge management the process consists of determining the rate of knowledge obsolescence, staff knowledge assessment, identifying of knowledge distribution channels, development of measures to intensify the processes of knowledge diffusion, planning and continuous monitoring of the level of knowledge of employees.

To evaluate the speed at which knowledge becomes obsolete in a company we suggest by the rate of knowledge obsolescence introduced above. Calculating this factor requires quantitative measurement of the results of generation of applied and theoretical knowledge in the subject area of the company, for example, using metrics such as the number of papers, patents, frequency and scale renovation work tools, and so on. In fact, this indicator will demonstrate how to "subside" knowledge employees over time.

Assessment and monitoring of knowledge level in a company can be conducted with the tools of analysis of knowledge growth and accumulation proposed in the "The model of knowledge accumulation by individuals" part. Technically the assessment of staff knowledge is implemented by introduction of formalized professional competence and evaluation procedures, which have become quite traditional to high-tech companies. The proposed tool in knowledge management practice allows reasonable determination (quantifying) the shares of the knowledge in the total possible amount that will match the grades of competence introduced in a company. This approach helps to identify statistically generalized scenario of knowledge accumulation by employees, which, in our opinion, is adequate information (basis) for predicting the state of company's intellectual resources.

The main motive for knowledge evaluation of an employee is to receive response to the actual questions of knowledge management - what is the current level of knowledge of an employee? (and derived from this question: for example, what kind of problems/situations are they able to solve the company?) and how much do individual and general knowledge obsolete during time periods? (i.e. forecasting). In practice, we propose to implement forecasting the level of knowledge with the help of formula (12), that takes into account the trends of individual knowledge growth and knowledge evolution itself. The result will be the actual level of knowledge of individual employees in a given unit of time and, as a consequence, the general level of knowledge of a company. Or, in terms of professional competences: when the current professional category of an employee will move to the lower level? and for the general level of knowledge in a company – what would be the composition of professional competences of its employees after certain time?

Conclusion and Future Study

Considering the evolution trend of knowledge, we rely on the development trends of science as such, and which is more important, the trends in the development of its applied directions. First of all, it tends to formulate new questions, problems and tasks, dictated by problems of industries and needs of modern companies. Understanding this we focus on the further development of the presented toolkit and as the possible directions of its improvement we see:

- Improvement of the knowledge diffusion analysis tool by including competencies and grades attributes of employees which will allow to predict the state of the knowledge spreading network and its capacity in the context of various scenarios of knowledge management in a company;
- Development of tools to define indicators that will allow to identify the lag of the current level of knowledge in a company from general industry knowledge, and expanding the application possibilities of the tools presented in (Bakanova, 2015; Bakanova et al., 2015).

Disclosure statement

No potential conflict of interest was reported by the authors.

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